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THE TEACHING OF VERBAL PROBLEMS.

BY E. R. BRESLICH.

Verbal problems in algebra are usually regarded by the pupil as the most difficult part of the course. This difficulty arises out of a number of causes. The ideas involved are sometimes remote from the pupil's experience and are therefore of little interest to him.

Since various types of verbal problems, each calling for a special method of attack, are usually given in the same set of exercises, the difficulty is increased. If, as it frequently happens, these problems are given at the end of the chapter, there is a tendency on the part of the teacher toward passing over this work hurriedly, especially when more than the available time has been spent on the early part.

The information being given in words, and not in symbols, is not as easily classified under familiar mathematical categories as much of the formal work of algebra. For example, let it be required to write the expression

$$\sqrt{\frac{36a^3}{34b} \cdot \frac{32c^{-2}}{24a} \cdot \frac{9b}{8c^4}}$$

in the simplest form. Although at the first glance, the problem as a whole might impress the pupil as difficult, he will readily see upon closer analysis that the symbols suggest a number of things he may do. He may multiply the fractions under the

radical sign, or reduce them to the simplest form. He may square some of the numbers as indicated, or extract the square roots of some of the factors. The negative exponent might suggest a law by which he may change the power into one with a positive exponent. It will be to his advantage to decide which of these operations better be performed first to get the simplest solution of the problem, but any one of them will enable him to get a start. There is nothing in the statement of a verbal problem to suggest to the pupil a method of procedure. The detailed analysis which must be made before the pupil can derive the equation is difficult to make, and takes time and patience. It is a type of activity which does not appeal to many pupils of high-shool age as much as the manipulation involved in formal work.

However, the principal reason for the apparent difficulty in the verbal problems is the lack of a good technique for attacking and solving them. This is usually not as definitely worked out as for the formal work. Without such a technique the pupil loses much time while he is wondering what he should do, and when he finally succeeds in working one problem he does not necessarily know how to apply his experience even to another problem of the same type.

The fact that pupils spend a large amount of time trying to find a method for solving a verbal problem has sometimes been interpreted to mean that verbal problems require more real thinking than formal work. The trouble with the abstract work is that it is nearly all technique but lacks application. It therefore involves no functional learning. On the other hand, problem work gives much opportunity for the development of mathematical adaptability, but frequently lacks the necessary technique. With a technique directing his thinking similar to that of the formal work the solution of verbal problems may be made a matter as simple, or even more simple than most of the formal work.

When it is considered that in the ordinary textbook on mathematics about one third of the space is given to verbal problems, the need for a good technique becomes still more apparent. Some writers recognize the importance of functional learning involved in problem work have advocated a further increase in

the number of verbal problems, and it is possible that future books may include an increasingly larger amount of this type of work. The technique of teaching verbal problems should therefore be made an object of careful study on the part of every teacher of mathematics.

The pupil knows, of course, that in every verbal problem there is an unknown number, the value of which is to be determined by solving an equation. The difficulty usually lies in deriving this equation. The solution of the equation is then mostly a simple matter.

The pupil therefore needs some general directions showing him how to begin in every case, and some additional particular suggestions which he may apply to particular types of verbal problems. These directions should be so definite that the teacher should get immediate action from every pupil the moment a problem is assigned to him. Furthermore, whenever a problem is explained to the class, the suggestions should help to make the explanation so clear that no pupil will fail to profit by it.

However, it must be mentioned that there is a real danger in using too much of this type of work. The directions given to the pupil should be such that he may understand the method clearly and so that the teacher knows that he sees the process. But they should not be emphasized so much that the pupil acquires the habit of having all his adaptations made for him. There must be a great deal of problem work for which the pupil must develop his own technique. This should be done in the class room with frequent consultations with the teacher. Unless the pupil is allowed ultimately to make adaptations alone without further outside help he will become helpless and will not develop adaptability.

On the other hand, just as in the formal work certain types of situations occur so frequently that the teacher is justified in deliberately having pupils memorize certain formulas; he may set up a framework for the solution of certain verbal problems and drill the pupils in its use, thereby relieving them of a good deal of mechanical detail.

We shall first formulate some suggestions which will help the pupil with a particular problem.

In many verbal problems the equation is obtained by translating the statement word for word into algebraic language. The following is a typical example.

The ratio of two numbers is a. If the first is increased by m and the second decreased by n the ratio of the results is $3/2$. What are the numbers?

Using one unknown, the solution is as follows.

Let ax be one number, and bx the other.

Then $ax + m$ is the first number increased by m and $bx - n$ is the second number decreased by n.

Translating into symbols the statement "the ratio of the result s is $3/2$," we have the equation

$$\frac{ax + m}{bx - n} = \frac{3}{2},$$

which may be solved for x. Then ax and bx are the required numbers.

Using two unknowns, we proceed as follows.

Let x be one number and y the other.

Translating the statement of the problem, we have

$$\frac{x}{y} = \frac{a}{b} \quad \text{and} \quad \frac{x + m}{y - n} = \frac{3}{2},$$

The values of x and y are then obtained by solving this system of equations simultaneously.

The pupil's attention should be called to the useful device of representing in terms of one letter two or more numbers whose ratios are given. For example, two numbers in the ratio $4/5$ may be denoted by $4x$ and $5x$. Three numbers in the ratio $4:5:6$ may be denoted by $4x$, $5x$, and $6x$, respectively.

The facts involved in verbal problems are usually related to each other. If the pupil is made familiar with these relations, he will be able to derive some of the facts from the others. The motion problem is the most common of this type. Here we have the relations $d = rt$, $r = d/t$, $t = d/r$. By means of those relations the pupil can obtain any one of the three elements provided he knows the other two. He should therefore know the necessity and advantage of taking this step. Clearness can be attained by arranging the work in tabular form. To illustrate

the method, let us consider in detail the solution of a problem of this type.

A man travels to a certain place at the rate of 18 miles an hour. He returns on a road 6½ miles shorter, and makes the trip in 29 minutes less time, although travelling only 17 miles per hour. How long is each road?

The pupil may arrange the facts involved in a double column table, one for each road, as follows:

Elements.	Long Road.	Short Road.
d	x miles	$x-6\frac{1}{2}$ miles
r	18 miles per hour	17 miles per hour
t	$x/18$ hours	$\frac{x-6\frac{1}{2}}{17}$ hours

The first element d is denoted by x because it is the unknown in the problem. The second element r is given. The third is derived from the first two by means of the equation $t=d/r$. Not before he has completed this table, should the pupil try to state the equation. The 20 minute difference in the time, which has not been used in the table gives the equation. Since the hour is the time unit, the 20 minutes must be changed to a fraction of an hour. The equation is therefore

$$\frac{x}{18}=\frac{x-6\frac{1}{2}}{17}+\frac{1}{3}, \quad \text{not} \quad \frac{x}{18}=\frac{x-6\frac{1}{2}}{17}+20.$$

A study of the difficulties involved in this type of problem has shown that pupils are usually not successful in deriving the equation if they fail to state first all three of the facts in the table, and that many fail to recognize the necessity of using the same unit of time throughout the equation. These two points should therefore be emphasized in the teaching of verbal problems.

This technique may be used in a surprisingly large number of verbal problems found in textbooks on algebra. The following table is a selection of the best known types whose solution may be simplified by the method just described.

The scheme of tabulating the elements involved in a problem may be used with advantage in some problems in which there is no equation expressing directly a relation between the elements. The so-called work problems and loss of weight problems are

Type of Problem.	Elements.		Relation
Earning money	Number of days,	n	$p = rn$
	Pay per day,	r	
	Total pay,	p	
Selling and buying.....	Number sold,	n	$p = rn$
	Price of each,	r	
	Total price,	p	
Gain and loss	Purchase price,	p	$s = p + g$
	Gain,	g	
	Selling price,	s	
Interest, discount	Principal,	p	$i = (r.p.t.)/100$
	Rate,	$r/100$	
	Time,	t	
	Interest,	i	
Leverage	Arm,	a	$l = af$
	Force,	f	
	Leverage,	l	
Carriage wheel	Distance,	d	$d = nc$
	Circumference,	c	
	Number of revolutions,	n	

typical examples. In the first the elements are the number of days, the amount done in one day, and the amount done in a given number of days.

Illustrative Problems.

1. A and B together can do a piece of work in 12 days. After A has worked alone for 5 days, B finishes the work in 26 days. In what time can each alone do the work?

The following table gives the facts.

	A.	B.
Number of days it takes to do the work alone.....	x	y
Part of whole done in one day.....	$1/x$	$1/y$
Amount done in 12 days.....	$12/x$	$12/y$
Amount done in 5 days.....	$5/x$	....
Amount done in 26 days.....	...	$26/y$

The equations are obtained by expressing in several ways the whole piece of work. Thus we have

$$\frac{12}{x} + \frac{12}{y} = 1 \quad \text{and} \quad \frac{5}{x} + \frac{26}{y} = 1.$$

2. If 19 lb. of gold and 10 lb. of silver each lose 1 lb. when weighed in water, how much gold and how much silver is contained in a mass of gold and silver that weighs 80 lb. in air and 72½ lb. in water?

	Gold.	Silver.
Amount in mass	x	$80 - x$
Loss of weight of 1 lb.....	$1/19$	$1/10$
Loss of weight of several lb.....	$x/19$	$(80 - x)/10$

The equation expresses the sum of the partial losses as equal to the total loss:

$$\frac{x}{19}+\frac{80-x}{10}=7.5.$$

The following examples show in detail arrangements of the steps involved in the solution of a number of typical verbal problems not mentioned above. These arrangements have been found to be helpful to the teacher in presenting the problem and to the pupil in finding the solution.

1. A man has two sons, one six years older than the other. After two years the father's age will be twice the combined ages of his sons, and six years ago his age was four times their combined ages. How old is each?

Let x be the number of years in the father's age.
Let y be the number of years in the age of the younger son.
Then $y+6$ is the number of years in the age of the older son.

After two years $\begin{cases}\text{the father's age is } x+2 \text{ years,}\\ \text{the younger son's age is } y+2 \text{ years,}\\ \text{the older son's age is } y+8 \text{ years.}\end{cases}$

Hence $x+2=2(y+2+y+8)$.

Six years ago $\begin{cases}\text{the father's age was } x-6 \text{ years,}\\ \text{the younger son's age } y-6 \text{ years,}\\ \text{the older son's age } y \text{ years.}\end{cases}$

Hence $x-6=4(y-6+Y)$.

2. The tens digit of a number of two digits is 6. If the order of the digits be reversed, the resulting number is 27 greater than the original number. Find the number.

Before taking up the solution of this problem the teacher should tell the class something about the principle of position in arithmetic. It may be shown that an arithmetical number, such as 256, is a brief way of writing $200+50+6$, or $2.100+5.10+6$. A number of three digits in algebra cannot be expressed in as simple a form as 256, since it is agreed that $x\ y\ z$ should mean a product of the three factors x, y, and z. Hence, the longer form must be used, giving the number $100x+10y+z$. If the digits are reversed the resulting number will be $100z+10y+x$.

Although the unknown called for in the problem is the required number of two digits, it is convenient to denote the unit digit by x.

Then the number is $6.10 + x$.

The number resulting by reversing the digits is $x.10 + 6$.

Hence $10x + 6 = 27 + (60 + x)$ is the equation from which the value of x is determined. Having found the value of x it is a sample matter to find the number $60 + x$.

3. How much water must be added to 12 gallows of a 25 per cent. solution of alcohol and water to reduce it to a 10 per cent. solution?

Let x be the number of gallons of water to be added. Then the facts may be arranged as in the following table.

	Old Solution.	New Solution.
The amount of alcohol and water	12 gallons	$(x + 12)$ gallons
The amount of alcohol...	25/100 12 gallons	10/100 $(x + 12)$ gallons

Since the amount of alcohol has not been changed in diluting the solution, we have the equation

$$\frac{25}{100}(12) = \frac{10}{100}(x + 12).$$

4. The time of revolution of Venus about the sun is about $7\frac{1}{2}$ months, and that of Mercury 3 months. How many months after Mercury is in the line between Venus and the Sun will it next be in the same relative position?

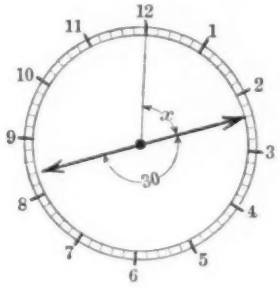
	Venus.	Mercury.
The required number of months	x	x
Time of revolution.....	$7\frac{1}{2}$	3 months
Part of orbit passed over in 1 month	1/7.5 of a revolution	1/3 of a revolution
Part of orbit passed over in x months	x/7.5 of a revolution	x/3 of a revolution

Since Mercury makes one revolution more than Venus, we have the equation

$$\frac{x}{3} = 1 + \frac{x}{7.5}.$$

5. At what time between eight and nine o'clock are the hands of a clock opposite to each other?

Since the time is given by the number of minutes after eight o'clock, let x be the number of minute spaces passed over by the minute hand since eight o'clock.



As the minute hand passes over 60 minute spaces, the hour hand passes over five, *i. e.*, over 1/12 as many as the minute hand.

Therefore $x/12$ is the number of minute spaces passed over by the hour hand since eight o'clock. The equation may be obtained from a drawing. Thus, we may express the distance from 12 to the hour hand in minute spaces. This gives the equation

$$x + 30 = 40 + \frac{x}{12}.$$

We may also express the distance from 12 to the minute hand, giving the equation.

$$x = 10 + \frac{x}{12}.$$

In the last equation use is made of the fact that the distance from 2 to the minute hand is the same as the distance from 8 to the hour hand, which is based upon the equality of the two opposite angles at the center.

6. *Geometric Problems.*—The relative number of verbal problems of geometric content is gradually increasing not only in the newer textbooks, but in the revisions of the older ones. The equation in this type of problems usually expresses some geometric fact, such as the sum of the angles of a triangle is 180 degrees, the area of a circle is r^2, the volume of a rectangular

box is the product of the three dimensions, and the square on the hypotenuse is equal to the sum of the squares of the sides.

As a rule these problems are not very difficult because the equation is easily derived by translating the geometric principle into algebraic symbols.

The following simple directions apply to all verbal problems and aim to help the pupil to get an immediate start.

1. *Read the problem carefully.* The problem may be read silently by all pupils, or aloud by one pupil. The purpose of this is not primarily to teach pupils to read accurately. We know that some have difficulty in reading. It may be possible and desirable to test pupils from time to time throughout the course to see whether or not they develop in reading ability, and to give them training in reading as applied to the mathematical field. The suggestion that the pupil should read the problem carefully is here given with the purpose of breaking pupils of the habit of attempting to solve a problem without first getting clear notions regarding the facts involved. This first reading is to get the pupil started and to give him a general idea of the content. Pupils should not try to study particular parts of the problem at this time.

2. *By further reading find what the problem asks for.* This will help the pupil to determine the unknown number, or numbers.

3. *Let x denote the unknown number.* Any other letter, of course, may be used as well. Emphasis must be placed on a definite statement in writing. Thus, if the rate is the unknown, the pupil should write: "let r be the number of miles per hour," not "let r be the rate." If the price is called for, the statement should be: "let n be the number of cents," not "let n be the price." In this manner the pupil will cultivate the desirable habit of always defining the letter before using it in an equation. Although, as a rule, the pupil should denote by a letter the unknown called for in the problem, there are exceptions. The solution in some cases may be simplified by choosing some other number as the unknown. The following problem illustrates this: "The denominator of a fraction is 5 greater than the numerator. If 1 is added to the numerator, the value of the resulting fraction is $\frac{1}{2}$." The problem contains three unknown

numbers, the numerator, the denominator, and the fraction. Denoting the numerator by n, the denominator is $n+5$, the fraction is $n/n+5$, and the equation takes the simple form

$$\frac{n}{n+5}=\frac{1}{2}.$$

4. *Read the problem again, one sentence at a time, and express the various facts involved in terms of the unknown literal number.* For example, let the first sentence read: "A sum of $5,330 is to be divided into two parts." The two parts may be denoted by two letters such as x and y. Then the first sentence will then read $x+y=5,330$.

If only one unknown is to be used, the pupil should write these facts as follows.

Let x be the number of dollars in one part.

Then $5,330-x$ is the number of dollars in the other.

If the teacher insists upon these four steps in the solution of all verbal problems he will train the pupil continuously in expressing in terms of algebraic symbols facts given in the form of verbal statements. Moreover, every pupil will at least get a start which usually results in a successful solution of the problem.

Experiments have shown that efficiency in solving verbal problems is attained more rapidly by individual effort of the pupil than by explanation of the solutions of a large number of problems. A new type of problem should first be presented by the teacher, emphasizing the technique as he works out the solution. A model solution should then be written on the blackboard. Next individual assignments should be made and help should then be given only after the pupil has made serious effort. A problem should be explained to the class only after every pupil has done considerable work on the problem. Explaining problems which the majority of pupils in the class have never tried to solve involves a relatively great waste of time.

Since the solution of the equations obtained from verbal problems usually offers no difficulty, many of the verbal problems may be worked only to the point of stating the equation, or equations. This will give a large amount of training in translating verbal statements into algebraic symbols.

Where solutions are worked out completely, they should frequently be checked in the original problem, because the roots of the equation do not necessarily satisfy the conditions of the problem. The following two examples illustrate this.

1. Find the length of the altitude of an equilateral triangle in terms of the side.

Denoting the length of the altitude by h and the length of the side by a, we have the relation

$$a^2 = h^2 + \frac{a^2}{4},$$

from which we find that

$$h = \pm \frac{a}{2}\sqrt{3}.$$

Evidently only $h = +2/2\sqrt{3}$ satisfies the condition of the problem.

2. According to the law of falling bodies, the time t, in which a ball thrown upward with a velocity of 100 ft. per second reaches a height of 144 ft., is given by the equation $144 = 100t - 16t^2$. Find the value of t.

The values of t satisfying this equation are $2\frac{1}{4}$ and 4. Both of these clearly satisfy also the condition of the given problem.

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THE TEACHING OF MATHEMATICS IN THE JUNIOR HIGH SCHOOL.

By Margaret Elsie Davis.

The phase of the J. H. S. movement which is of vital and immediate interest to us is the teaching of mathematics in this school unit. However, the problems that require consideration in connection with this branch of the curriculum are necessarily related to the problems of the whole movement and must be solved with due regard for that relation. Among the problems to which I refer are those of aim, subject-matter, method, sequence of subjects and the time element.

The present attitude toward and conception of these problems have been in the forming since the later eighties at least. They are the "result of dissatisfaction with (1) the existing rigidity of the grade system, (2) the lack of economy in education, (3) the failure to relate school work to social and economic conditions of life and (4) the failure to recognize and to provide for the changing conditions in the physical and mental characteristics of those to be educated, particularly during the period of adolescence."

To overcome these defects the type of school known as the Junior High School has been evolved. An examination of the ideas of the J. H. S. which are held by the most prominent educators shows that the "principle of individual differences is emphasized more than any other; but other principles regarded as vital are brought out: namely, the reorganization of subject-matter from a social standpoint and its placement upon a sound psychological and pedagogical basis for instruction."

These and minor principles when applied to and executed in the mathematics course of the J. H. S. should result in the following conditions:

Mathematics work

(1) which is suitable for all pupils approximately 12 through 16 years of age

(*a*) who will remain in school and pursue elective mathematics in the Senior H. S., and

(*b*) who for various reasons will assuredly leave school early, and who ought to be given some insight into mathematics of a varied and a vocational character;

(2) which reveals the pupils' interests, aptitudes and capacities by the use of mathematical material in itself worth while;

(3) which open up the mathematical possibilities in the senior subjects and in several industries of local importance;

(4) which provides for individual mathematical differences by differentiated curricula and by the organization of groups homogeneous in ability;

(5) which uses methods intermediate between those of the elementary and the high school;

(6) which provides for specially prepared teachers of mathematics through the partial or full use of the departmental plan;

(7) which uses subject-matter that will meet immediate and certain assured future needs along social and economic lines.

SUBJECT-MATTER OF THE MATHEMATICS COURSE AS A WHOLE.

One of the best general statements regarding the principles underlying the reorganization of the mathematics course of J. H. years was presented by the National Society for the Study of Education in the 15th Yearbook—Part III. It specifies that in the place of arithmetic, algebra and geometry, which represent a logical and not a psychological sequence, a course in mathematics should be substituted which represents a unification of these three subjects after certain parts of each have been eliminated. Arithmetic will perhaps form the bulk of instruction in the seventh and the first half of the eighth grade. The last half of the eighth year and the first part of the ninth will consist mainly of algebra, with special emphasis upon the equation, but at the same time the facts of geometry applicable to the work in algebra could well be given. Toward the end of the ninth year the work will be mainly geometry.

A study of the various recommendations of educators and of course of study indicates a general tendency to incorporate in the body of subject-matter,

(1) arithmetic (largely of the social and economic type),
(2) experimental or constructional geometry,
(3) algebra,
(4) trigonometry (elements only),
(5) graphs,
(6) demonstrative geometry (variable from none to one-half year's work).

Several prominent educators favor differentiated courses, *e. g.*, shop and business mathematics, in the ninth year.

Methods of the Mathematics Course as a Whole.

The aims and principles, previously stated, should determine to a very great extent the method, and therefore, the vitality and appeal of the course.

The methods of the past against which a reaction has occurred were logical rather than psychological. To a large extent they were systematic. Topics were introduced by an elaborate treatment of definitions and rules which meant little or nothing to pupils because they did not arise through a need for them nor was there any immediate application of them. Naturally very little interest accompanied such methods which lacked meaning for the pupils.

Insofar as the technique of past methods was good and at the same time made it possible to arouse interest and to provide motive and application in its desirable to retain it.

The project method, so widely advocated and used at the present time, develops the subject in connection with carefully chosen problems. It emphasizes the whole-hearted purposing of the doer and thereby involves the best of the modern tendencies in education. It is excellent as a means of arousing thought and preparing a student to solve the mathematical difficulties of life experiences. However, if used exclusively, it is likely to prove too slow. I may exclude the necessary drill which is so essential.

The method should be dominated by understanding, appreciation and skill. New principles should be introduced by means

of simple, concrete exercises and be consciously developed. The processes should be treated rationally. The individual should observe and think for himself. The presentation should be followed by drill exercises to secure a satisfactory degree of skill and by suitable applications to insure appreciation of the principle being taught.

A moderate amount of supervised study is desirable in order that pupils may have the advantage which such work offers, particularly that of meeting individual needs.

Correlation of the right kind should be employed. The various branches of mathematics are not to be taught independently of one another. The early introduction of geometry is a great aid in making clear algebraic processes and principles. As far as possible all the important facts and principles of mathematics should be kept under review in successive, related topics, to insure a greater skill in the use of them and an increased probability of their permanent retention. Correlation should be understood to mean unification and clarification, not fusion, which so often results in confusion.

The mechanical juggling of symbols and figures is discouraged. The teaching should not be primarily a text-book affair. The J. H. S. calls for more mathematics and less text-book and symbolism.

The motivation needs to be of the strongest kind. The idea embodied in this term is very important. It is regarded by some educators as the chief hope of improvement in J. H. S. mathematics.

In general, it may be said that the method is such an important item that, through its improvement only will mathematics thrive and survive.

Subject-matter of Arithmetic

The fundamental changes in subject-matter proposed for arithmetic are the elimination of or reduction of time to be given to certain topics and the increase of time or emphasis to be given to others because of their social interest.

Mr. W. A. Jessup has prepared a table based upon a questionnaire sent to about 1,700 city superintendents and to every sixth county superintendent in the U. S. The majority of these

superintendents favor elimination of or less time to be given to apothecaries' and Troy weight, such measures as furlong, rood, dram, surveyors' tables, foreign money, reductions of more than two steps, G.C.D., L.C.M., true discount, cube root, partnership, compound proportion, compound and complex fractions, certain cases in percentage, annual interest, longitude and time, unreal fractions, metric system, progression and aliquot parts.

They favor more attention to the fundamental processes, fractions, percentage, interest, saving and loaning, banking, borrowing, building and loan associations, investments, stocks and bonds, taxes, levies, public expenditure, profits and utilities.

Closely allied with this question of elimination is that of the time to be devoted to arithmetic. An examination of various curricula shows an average difference of about one half year. *E. g.*, the National Society for the Study of Education places arithmetic in the seventh and first half of the eighth grade; the Rochester plan assigns it to the seventh year; and the ethical culture plan devotes only part of the seventh year to arithmetic and the remainder to intuitive geometry.

The topics for which more time is advocated by the superintendents to whom reference was made are fairly representative of those holding a place in the majority of the J. H. S. curricula.

Methods of Arithmetic.

The arithmetic work assigned to the J. H. S. involves practically no new processes and principles. The new work is largely informational in character and its source lies in the social and business conditions indicated by the topics outlined in the subject-matter.

The applications of percentage should be introduced by means of parallel types of problems in common and decimal fractions and percentage.

Thorough reviews and drills in the fundamental processes, fractions and decimals should occur as a consequence of this method of treating the new work. Also, the fundamental unity of arithmetic is emphasized, the work is made clearer, and time is economized.

The arithmetic must be correlated with the lives of the pupils

by finding genuine applications of the work that are really within their experience and life activities.

As an aid in analysis the unknown term and the equation from algebra are introduced wherever needed.

Mensuration and other topics of measurement should be facilitated by the introduction of constructional geometry.

Constructional, Observational, or Intuitive Geometry.

Mr. J. C. Brown's summary of the world's curricula in mathematics as outlined in reports of the International Commission states: "Some instruction in constructional, observational or intuitive geometry is always offered during the sixth, seventh, and eighth school years. This instruction is always of a propædeutic nature. Much emphasis is placed upon estimates and constructions.

"In all of the schools of Europe algebra and geometry are studied simultaneously during a considerable number of years. The various mathematical subjects are more closely correlated than in this country. A pupil who is studying geometry can use his arithmetic and algebra more readily than is the case with the average American boy. The introduction of the trigonometric functions while the pupil is studying similar figures in geometry has the sanction of most of the best teachers abroad. The distinction between plane and solid geometry is much less marked than in this country. This is due, in part at least, to the fact that models are very extensively used in the study of geometry."

The consensus of opinion of mathematical educators is that geometry should be begun as early as possible and certainly before algebra. Mr. William Betz maintains that it should be possible in the course of time to provide for some geometric instruction in the seventh grade, where it belongs.

Historically, this sequence, *i. e.*, arithmetic, geometry, algebra is the only correct one and experience verifies this.

Intuitional or constructive geometry is more concrete than algebra and admits of more simple illustration.

The algebraic formula may be gradually introduced in connection wtih the geometry work, so that the student may be made aware of the value of algebra as a working tool before he proceeds to the study of algebra.

Subject-matter of Geometry.

The subject-matter of geometry may be grouped under the geometry of (1) form, (2) size, and (3) position. The majority of courses include part of all of the following:

1. Historical introduction.
2. Classification of forms.
2. Study of
   - lines,
   - angles,
   - triangles,
   - other polygons,
   - circles.
4. Use of drawing instruments:
   - slide rule,
   - compass,
   - protractor,
   - right triangle,
   - ruler.
5. Construction of
   - angles,
   - triangles,
   - perpendiculars.
6. Bisecting
   - lines,
   - angles.
7. Parallels.
8. Drawing to scale.
9. Accurate proportions.
10. Congruence.
11. Equality.
12. Similarity.
13. Symmetry.
14. Geometric measurements:
    - lines,
    - surfaces,
    - solids.
15. Simple surveying.
16. Ratio and proportion in connection with similar figures.
17. Locating points by lines and angles.

Methods of Geometry.

General principles of method were stated for the entire course in mathematics. A few details of the method follow.

The work at first is entirely observational and intuitive, but by low degrees rational processes are introduced.

Thoroughness of treatment is not sacrificed to breadth of treatment.

In whatever grade the geometry is begun it is advisable to give a preview of the course lasting two or three weeks. The pupils should get an idea of what geometry is, not through a dissertational course but through a concrete introduction. The geometry should not begin with projects but rather with evolutionary views.

Pupils should be shown how geometry originated through measurement as arithmetic did through counting. (McLellan and Dewey give a thorough treatment of this in "Psychology of Number.")

Projects may be brought in by asking, "Why did people count? Why did they measure?" Through these projects the geometry involved in (1) shelter, (2) clothing and (3) food may be discussed.

1. Shelter: Refer to the building of primitive houses, *e. g.*, the hemispherical home of the Esquimaux and the form of the Indian hut. Use these forms in the room and lead the children to see that through the activity of house building we get fundamental forms.

2. Clothing: Present looms used in weaving. Notice that the warp and woof give parallel lines, squares, etc.

3. Food: The primitive activity of agriculture gave geometric forms. Ask why so many fields are rectangular, why the rectangle is the form of practical geometry and the triangle of theoretical geometry.

Show examples of ornamental tatooing, Indian basketry and weaving.

The purpose of such a course is (1) to interest pupils in forms, (2) to get pupils to draw forms, and (3) to develop geometric ideas.

Through the study of ornamental objects lead the pupils to an appreciation of the four principal elements of geometry, *i. e.*,

(1) equal spacing, (2) symmetry, (3) congruence, and (4) similarity.

Lead the pupils to discuss the school room as a laboratory for geometry. Ask why the room, doors, windows, etc., are rectangular and the subsidiary forms have the same shape as the principal form of the room. Parallel lines exist and are important everywhere. Lead the pupils to look for them in nature, art and industry.

Note the symmetry of the butterfly and other natural objects.

Encourage pupils to prepare a geometry museum as an equipment to motivate the work.

Use pictures that represent designs of buildings.

Use slides loaned by the State Department of Visual Instruction to illustrate geometrical forms seen in the Egyptian structures, the Parthenon, etc.

Visit a house of industrial photography.

Have geometric exhibits in school.

Such exercises as the above permit of "geometry's working its way into children, since it cannot be taught into them."

Algebra Subject-matter.

This includes practically all of the topics usually treated in elementary algebra, *i. e.*,

1. Notation,
2. Symbols,
3. Formulas,
4. Equations,
5. Graphs,
6. Negative numbers,
7. Addition, subtraction, multiplication and division,
8. Fractions,
9. Factoring,
10. Equations, simple and quadratics,
11. Methods of checking,
12. Square root and Pythagorean theorem.

Pupils should be familiar with some of this subject-matter through its use in arithmetic and geometry. They may have some idea of (1) the nature of algebra and of its importance through their work with formulas, graphs and equations, (2)

the use of negative number, and (3) the applications of algebra to measurements of various kinds and in the solution of arithmetical problems.

Methods of Algebra.

In the past algebra has been taught altogether too often as a symbol-juggling course. Needless to say there must be a decided reform along this line in order that children may catch the spirit of the subject. The pupils in the J. H. S. algebra classes represent future artists, carpenters, some Senior H. S. and college candidates, etc. It would not permit the J. H. S. to fulfill its function if algebraic methods were such as to drill all the pupils until they attained "professional proficiency in factoring unusual expressions, removing ingenious nests of parentheses and simplifying a marvelous complexity of radicals."

The method should emphasize algebra as the shorthand of science. *E. g.,* consider the statement of the area of a rectangle. The generalization in longhand form is the complete sentence, "The area of a rectangle equals the product of the length and the width." But since it is a fundamental tendency of the mind to economize we may "economize" the above statement thus, $A = lw$.

The formula work may be motivated by a discussion of the ways in which we economize by shorthand in life, *e. g.,* commercial shorthand uses the initials U. S. A., C. O. D., a person's own initials.

The pupils may be asked to name various kinds of quantities that involve counting and measuring and to list these according to the branch of industry from which they come, *e. g.,*

Businesses	*Mensuration*	*Science*
principal	length	watt
percentage	width	pressure
interest	height	resistance
discount	distance	ohm
profit	radius	ampere
loss	circumference	

The field in which the pupils will probably find the greatest number of words is mensuration. This field is good for illustration of the point, *i. e.,* shorthand expression of terms by use

of initials, since it affords opportunity for *showing* things and makes a real background to which to attach symbolism.

Point out the use of shorthand for expressing the measurement of geometric figures, *e. g.*, 4*s* (sides) expresses the perimeter of the square.

The method should show clearly that algebra is a tool for solving applied problems. This is done through a study of the equation. Ask the pupils to consider the following problem. A coal bin, large enough to contain 8 tons of coal, is to be built in a certain cellar. The arrangement of the cellar requires the bin to be 6 feet wide and 6 feet high. What must be the length of the bin, allowing 36 cu. ft. for 1 ton of coal? First ask for a solution by use of the complete anaylsis method. Then ask for the formula for volume, and from this the equation which becomes $6.6.x = 288$, giving x a value of 8.

The superiority of the equation method becomes so apparent that pupils readily grasp its value.

Since the representation of generalizations by algebraic equations is considered as one of the most unique contributions to civilization, we cannot afford to neglect its emphasis in algebra.

The method of algebra should present clearly the function of the graph as a superior means of presenting statistics, etc. Contrast this with the table and descriptive methods of presenting facts, to make the advantage of the graphical method prominent.

Lastly, let me refer to the need of unifying all the mathematics work by correlating the algebra wherever possible with the arithmetic and geometry which precede it.

Trigonometry.

Many of the newer courses of study in mathematics include an introduction to the meaning of and the practical uses of trigonometry.

The subject is introduced with the tangent—the simplest and most natural trigonometric function, because of the variety and nature of its applications.

The use of the transit for measuring angles in outdoor work is advised.

There is a modern tendency to develop the idea of function in connection with the arithmetic work since problems in arith-

metic illustrate the great importance of functions in mathematics.

Because mathematics is so largely a science of function it is considered advisable to introduce this work earlier in the course. The name "function" and the definition of the term should be postponed until the idea is thoroughly established.

TEACHERS' TRAINING SCHOOL,
BUFFALO, N. Y.

THE METRIC SYSTEM. ITS RELATION TO MATHEMATICS AND INDUSTRY.*

By Wilmer Souder.

Mathematics as a Foundation.

Lord Kelvin has said that our understanding of a physical phenomenon is not complete until we are able to express its relations in the form of a mathematical equation. This is equally true in many other fields of activity. Even the smallest capitalist, the newsboy, is not getting all the pleasure (or worry) out of life unless he is able to accurately compare his gains and losses with those of former days and with those of his fellow workers. There is no more fundamental science than the science of mathematics, and I envy you, your position in our educational system. There is a definite yes or no to all your problems. The student who distorts the problems in any way must somewhere make a second error or inevitably arrive at the wrong answer. There is little or no occasion for an argument or back talk in your work. For the development of precision, definiteness, and confidence your subject must always remain at the top.

Success in Teaching.

As a measure of your success in this work you are in a large degree dependent upon the success of your students. The ideal measure would be to give each student an examination in applied mathematics after five years' experience in the world, then award to you the average per cent. grade as a true mark of *your* success.

My experience in teaching junior and senior students of physics at one of our largest universities has brought to my attention two examples of lack of emphasis upon certain phases

* Presented at the Spring Meeting of the Association of Teachers of Mathematics in New England, May 1, 1920.

of their college training. Since these will have a definite bearing on what I shall say later, I wish to take them up in detail at this time.

First: The absolute failure to comprehend the limitations of accuracy when applying laboratory data to mathematical equations causes hours of unnecessary work, *e.g.*, in the case of Young's modulus, where we have:

$$Y = \frac{F/A}{e/L} = \frac{FL}{Ae}.$$

It is difficult to measure the wire diameter to better than one. part in 100 and when the modulus is computed, it is not uncommon to find note books with 12 significant figures instead of the usual two figures by the power of ten. Shall we call the correction of this a disillusionment of the student's conception of the accuracy of mathematical formula or shall we simply say, this is the practical limitations of the associated science? Your answer to this question may be "used as evidence against you" in the subsequent developments.

The second defect lies in the inability to interpret mathematical formula or to define the conditions without using mathematical terms: To illustrate

$$Y = \frac{FL}{Ae} = \frac{F/A}{e/L}$$

may be defined as the ratio of the force per unit cross section to the elongation per unit length, a strictly mathematical definition and from such a definition the student should see at once that reducing the diameter ½ will increase the elongation 4 times provided all other conditions remain the same, or again doubling the length of the specimen will not change the value of Y. The physical interpretation may appear less definite but is not less important: Young's modulus for any material is numerically equal to the theoretical force required to stretch a uniform bar of unit cross section to twice its original length. Manifestly the realization of this definition is impossible, although the conception is quite definite.

Teaching the Metric System.

In our consideration of the metric system of weights and measures we must insist upon the above applications and limitations. You are not teaching the metric system when you ask a pupil to convert 1 mile to millimeters, nor when you ask a student to compute the cost of 1 kilogram of meat when priced at 50 cents per pound. This is a close mathematical analogy to the "cat skinning" exercises of the gymnasium, and while it may appear to have a certain value in our educational system, there are those who feel that the time could be spent for greater profit to the student.

We object when the delivery boy starts juggling *our* packages regardless of his expert abilities as a juggler. We admire the man who can play 8 simultaneous games of chess blindfolded but question just how long he would hold down a job as advertising manager for Quaker Oats if he persisted in displaying his abilities while handicapped by a hoodwink. So it is with many of the advocates of cumbersome weights and measures. There is no use to deny the fact that commerce and production can be conducted on the present plan; but should the handicap once be removed it would be difficult to find any one (except those who have rushed into print) who would favor a return to the old shackles and it is quite possible that some of these could even point out certain statements in their printed articles which would prove their insight as to the superiority of the metric system. Stranger things than these have happened.

The actual teaching of the system is so simple and brief that scarcely more than a page is necessary. Look up your arithmetics—eliminate all the problems of 11 yds. = ? mms. and 5 gallons = ? liters; all of which are not definite enough for solution and each of which is an apology for the English system. In no sense are these to be considered teaching the metric system. After eliminating all such, see how much is left, also how direct and to the point is the real metric part of the work. Educators claim that at least 2 years are lost by burdening our students with the old cumbersome system, not to mention the discouragement and despair of many unfortunates who conclude that because they are not adept in assorting a patchwork of grains, drams, scruples, stones, carats, ounces, pennyweight,

quarters, hundredweight, tons, pounds, etc., they have no mathematical ability.

By analogy I would feel like saying that the boy who can not turn cartwheels, handsprings, walk on his hands, jump stiff-kneed and do all the fancy dives, has not the ability to walk.

Teaching the metric system will automatically take care of itself. If you want approximate values, the kilometer may be thought of as a 10 minutes' walk; the meter as an arm's reach from the tip of the nose; the centimeter as a small finger's thickness; the millimeter as the thickness of a 10-cent piece (the correct gap for a Ford spark plug); the micron as the bust measure of a bacterium; the gram as $\frac{1}{5}$ of the mass of our 5-cent piece; the liter as the volume of the soldier's canteen; the cubic centimeter as a small thimble-full; and so on.

When you name any apparent difficulty in handling the system you immediately convict yourself of negligence of the fundamentals of notation, multiplication or division; for these three operations plus the recognition of the three units—meter, liter and gram—make up the entire subject.

The English System of Weights and Measures.

I shall not burden you with the tedious and complicated stories connected with the origin and development of the English system of weights and measures except to mention a few of the more important. It is generally admitted that they are not original but a modification of the systems used on the continent prior to their adoption in England. We are told that at one time 3 barleycorns made 1 inch and that 12 inches = some foot, and 3 feet = 1 yard (the length of a certain King's arm). Then in 1835 all measures were burned in a fire which destroyed the House of Parliament, after which the values were re-established as closely as possible, the fundamental unit being the bronze yard, the length of which is even today indefinite.

We must admit that these measures are romantically tied to the past and that a certain amount of patriotism and loyalty may be interwoven in their origin; but certainly they are no more ancient than the Jewish talent and have nowhere near the possibilities for a claim of divine origin which may attach to the talent.

But we find that the talent had different values for different tribes and consequently had to be discarded. Similarly we are told that the English system has suffered similar corruption, that there are 154 different units of length; and in the British corn trade alone, 200 different weights and measures. The textile and yarn situation has degenerated until it is a hopeless problem to attempt to follow all the deviations.

This subject will be treated again. Suffice it to say now, that either there is a fundamental reason for these confusions or that the English system is a failure—since the confusion has increased rather than subsided in favor of the system.

The Metric System.

The Metric System—the perfected decimal system—is the result of numerous influences. Chronologically, we find the decimal ratio first mentioned in the Book of Exodus where we are told the ephah = $\frac{1}{10}$ homer, and again in the tithe of $\frac{1}{10}$. The more recent developments date from the close of the eighteenth century and forced themselves upon us because, as James Watt wrote: "It is very awkward that the scientific results of various workers can not be compared." The nineteenth century's scientific developments compelled the adoption of this system among scientists just as the twentieth century's manufacturing and commercial development demand its use in efficient, progressive trade.

In 1790 Talleyrand, the French statesman, brought a bill before the French Legislature asking to have a commission appointed to investigate the possibilities of a simple and uniform system of weights and measures, designating the French Academy and the Royal Society of England as responsible for naming the commission. Because of the war situation the Royal Society failed to function and the problem was left to the French. The final outcome being a simple decimal system, without ambiguities and now confirmed by definite accurately calibrated standards copies of which are in the possession of, or available, to, every civilized nation of the earth. So perfectly have these standards been worked out in form, composition, and value, that should all traces of civilization be destroyed from the face of the earth, except the standard meter, and should a new

civilization develop, the examination and study of this meter alone would be proof positive that the former civilization was of the highest order. The commercial relations of the system are shown to advantage by the chart which you see on the wall before you. (Chart explained.)

If you wish the fundamental relation, possibly the following will be sufficient; at least it won for the metrologist, Michelson, the Noebel Prize. I refer to the exact value of the meter which was established as

$$1 \text{ meter} = 1\,553\,163.5\ \lambda \text{ Red. Cd. at } 15^\circ \text{ Nor. Bar.}$$

The red radiation of cadmium, or radiation from any element, has remained constant from the time of its discovery and is not to be considered in the class with arm lengths, foot lengths, toe widths or the distance at which one can see barleycorns.

Allow me to say at this time that the statements made in this talk are not to be considered new or original; they are in many cases a re-arrangement of previous articles and talks on this subject. Numerous writers have elaborated upon the subject and I have taken the liberty to select some of their ideas for presentation to your association.

Doubtless you have been wondering how I happened to come to Boston today. I shall be only too glad to explain. Your president asked Dr. Rosa, of Washington, to name some one to discuss the metric system before the N. E. Teachers' Association on May 1. There were three other matters to look after, in New York, Torrington, and Boston, at about this date, in which my work is concerned, so I am killing four birds with one stone, three for myself and one for your Association. I beg that you will not hold your president to too strict account.

It is because of the conviction that any step toward the adoption of the metric system is a move which will ultimately confirm its merits that I am pleased to review the situation with your organization. Boston holds a very advanced position in this activity. One of the pioneers, who thought the question out completely and advocated its early adoption, was the late Frederick Brooks of your own city. We are under obligation to him for his forceful and clear arguments in favor of these units.

The Confusion of Systems Now in Use (Other than the Metric).

There are over 150 different liquid measures in use in the world all different in name or value. There are over 100 units of mass. Sixteen of which have the same name with 14 different values for the unit, and so on with other units. Now standardization has always meant efficient production and a corresponding reasonableness in price. If you can find your fit in ready-made clothing, you are able to save a considerable amount of money. The instant you recognize a diversity of units or standards, you license the best chance for fraud. If you receive shipments from the 14 countries having different values for their pfund, the quickest and most likely plan will be to fix the selling price on the basis of the smallest mass, as the basic pfund, and who can convict the commercial agency of fraud, for isn't one pfund just as legal as another?

Until we can have uniformity and definiteness of units we shall have difficulties in regulating trade, either imports or exports. The English gallon is about 20 per cent. larger than ours. The English yard is probably not the same as ours. And our own pounds are different when you compare those used by the grocer, jeweler and druggist.

I want to give you a concrete illustration of the complications of the English system when used in its simplest form. The following problem is offered in regular form, free from catches and in units quite at home to all of you. To the first one giving the correct answer, I will present the chart which I have been using.

From 1 mile subtract

.7 furlongs, 39 rods, 5 yards, 1 foot, 4 inches.

(The solution of this problem required several minutes. Incorrect answers were reported in some instances.)

Objections to the System. Unfairness and Inconsistency.

We are told that the metric system is unsuitable because its units will not divide by 2, 3, 4, 6, etc., indefinitely; whereas the 12-inch unit will. If true, then we need 12 inches=1 foot; 12

feet = 1 yard; 12 yards = 1 something, etc.; also 12 ounces = 1 lb.; 12 lbs. = 1 stone; 12 stones = 1 cwt., etc.; 12 gills = 1 pt.; 12 pints = 1 gallon; 12 gallons = etc.; 12 cents = 1 dime; 12 dimes = 1 dollar, etc. This, of course, points to the duodecimal system with all its complications.

Again it is pointed out that the metric system is a failure, since it has not displaced all others because of its inherent superiority. Listen; Weeds always tend to grow in the best gardens and have to be kept down. The vice of this city could never be kept down by the perfect virtue of your ward. Law and law enforcement are necessary and will probably be advantageous for a number of years. Now allow me to point out the defection of the English system and at the same time to intimate that such has never come to the metric despite all the prejudice, ignorance, and inexperience in its use.

Are all gallons alike? In spelling and pronunciation, yes. In capacity, decidedly no.

Are all pounds alike? In spelling, yes; in mass, no.

The same answers apply to the yard, to the mile, the ton, the barrel, etc.

You never hear of a short meter, of a large liter, or light kilogram. The units have stood the gaff and proclaim their excellence, their uniformity and permanency for all uses.

Recently the following problem appeared in the *American Machinist* and was intended to reflect on the system. In converting a simple expression to the metric values there resulted a value which was expressed in lengthy form.

1 lb. per square inch (a very simple expression) = 0.07030954 kg. per cm.^{2}, a somewhat extended and incorrect number; but observe that the value was extended to include hundredths of a milligram! A more nearly correct value is 0.0703066. Now we are often reminded that it is a poor rule that won't work both ways and that sauce for the goose should be sauce for the gander. So lets back up on the problem by converting 1 kg. per cm.^{2} to pounds per square inch carrying the value to the equivalent of .01 milligram. We get

1 kg. per cm.^{2} = 14.22339839 lbs. per sq. in. (but this decimal part is not English and must be properly expressed in the usual units).

= 14 pounds (avoirdupois, not apothecaries).
3 avoirdupois ozs.
9 drams
5.1950 grains.

You may draw your own conclusions as to the more simple method of expressing the similar values, as to the sincerity and motives of one who will resort to such extremes to confuse his readers, and as to the weakness of the case when it becomes necessary to imply such ignorance and stupidity on the part of the readers.

Again we are told that metric values will introduce fractions in manufacture; this is not strictly true, for the fractions are already there. Take for example the innocent simple half-inch pipe thread tap. Watch closely, many have failed to locate the half-inch value. For this tap you use a 0.6875 inch drill and the two diameters used in thread notations will be (for the finished thread) 0.8257 inch and 0.7342 inch, whereas the metric values would be 17.46, 20.97 and 18.65 mm. respectively.

Have you ever heard the statement: "The inch is a most convenient unit"? I have. Very well, suppose we look for some visible manifestations. Is this an inch? (measures numerous articles in sight) or this? The values are 1 cm., 2 cm. and this 4 cm. and this 1 cm. Does any one see anything in this room which is 1 inch. Surely there must be something exactly 1, 2, or 3 inches in dimension if the unit is so convenient. (Nothing discovered with exact inch values.)

Most materials are manufactured on the basis of a template pattern or a "go" and "not go" system. This means that the model is made up and allows a certain tolerance. A tolerance of one thousandth of an inch is considered accurate work on many items of the shop. This would be expressed as 0.9995 to 1.0005 inch. The metric equivalents of these numbers do not look any more cumbersome nor any more vicious and are just as easily laid out on a measuring device.

Many manufacturers have been talked into believing that the system will play havoc with their production rate and consequently we have a list who insist that no changes be made. This is not unusual. There were many objections to the Pure Food Laws. England still objects to a decimal system of cur-

rency and a majority of the taxpayers universally object to the tax rates for the next year. I have in mind a person who objected to the Rural Free Delivery System of the Post Office Department because it gave a man a job at $600 per year, and another who objects to all vaccinations, X-rays, and surgery, saying that if the Creator wants any foreign things in the body, He'll see to it that they go there and that it is useless to try to violate His will by removing them.

We must not listen to the complaints of the chronic objectors nor delay progress because of an imaginary fear that something may happen if advance steps are taken by our civilization. The advance step often furnishes the cure for thousands of former difficulties.

Objection is often raised on the ground that the metric system does not adapt itself to our foreign trade, attempts have been made to show that the consuls and foreign agents of the various countries of South America object to its use. The *Decimal Educator* for December, 1919, clears up this polluted atmosphere very nicely by giving over 20 letters from these countries' representatives and in many instances they reveal deliberate misrepresentations on the part of the metric objectors. All are in favor of the system and practically all comment upon the success of the system.

Again we are told that the mental capacities are limited, that certain classes are of (low?) a fixed mentality and can comprehend 2, 3, 4, 6, and possibly 8 but not 10 or 100; *i.e.*, the division of $\frac{1}{10}$ or $\frac{1}{100}$ are not comprehended as is the case of $\frac{1}{8}$ or $\frac{1}{6}$, etc. I fail to grasp the point, I feel it is an unjust reflection and that 10 cents as $\frac{1}{10}$ of a dollar or 1 cent as $\frac{1}{100}$ of a dollar is even more definite than $\frac{1}{8}$ or $\frac{1}{6}$ of a dollar. Try short changing a customer 1 cent on the dollar and see if he comprehends 1 in 100.

Proofs of the failure of the system would be: A slurring or modification of the values of the standards, the rejection by *important* countries or manufacturers after trial or the evidence that computations are more laborious.

The objections remind us of a story told about the railroad yard master who was looking for a spare engine to move a train of cars. The first engine was of the largest battle-ship type and

as it stood near the cars said: "Too-heavy! too-heavy!—too-heavy!" The master passed to the next, a speedy freight engine which said: "Can't do it!—can't do it!—can't do it!" There being only one engine left (a little dinkey switch engine) he ordered it hitched up. The little fellow, quite willing to try, started up the grade saying "I—think—I—can!—I—think—I can! I think I can!' (and as it went over the hill changed to) "I thought I could! I thought I could! I thought I could!" and I dare say this will fit the manufacturing situation more closely than we think. The manufacturer's foremen are not deficient in brains, and the abilities shown in setting lathes and shapers for the innumerable complicated parts of machine guns, torpedoes and motors lead us to infer that they are not too stupid to apply a set of simple translation gears and scarcely notice the effort required.

The scientists have solved the difficulties and are universally using the metric system. No one questions but that it is the best for their work and that it is a success; that the language and constants of scientific research are universal. English and American scientists were able to step into the French War Laboratories and begin assisting in the work within the hour. Since a majority of your students will come in contact with the important scientific developments of the age; it is worth your while to make them experts in the metric system; if for no other reason than that they may be capable of comprehending and using opportunities of the scientific world.

Another rather queer objection which is often offered is a statement that the confusion in the English system is due to the attempt to adopt the metric, etc. Such objections arise from a too literal interpretation of Mark Twain's advice: "Get your facts first; later you can distort them as much as you please."

Let me close by comparing the confusion due to the metric system to the boy's definition of salt: "Salt is what spoils your potatoes when you don't have any on them."

WASHINGTON, D. C.

EDUCATIONAL OPPORTUNITY IN THE ARMY OF OCCUPATION.

By J. T. Rorer.

In the fall of 1918, the Y. M. C. A. sent out a call for two thousand educational executives. These men were selected from the faculties of normal schools, colleges, and universities. There was also a small representation from the public schools, generally men of extended experience as specialists, principals, or superintendents. Perhaps one third of the men were fortunate in securing leaves of absence from their regular positions, but the great majority were so enthusiastic over the patriotic service that thus opened before them, that they willingly resigned important posts to accept assignment in the A. E. F.

As rapidly as possible, the men (and a very few women) were sent in groups of ten to fifty to Paris where they reported at the General Headquarters of the Y for assignment. The placements were made by the Educational Commission which consisted of Professor John Erskine, Columbia University, Chairman; Dr. Frank E. Spaulding, Superintendent of Schools, Cleveland, Ohio; and President Kenyon Butterfield, of the Massachusetts Agricultural College. Professor Erskine took charge of the American University at Beaune, Dr. Spaulding of elementary and high-school work throughout the A. E. F., and Dr. Butterfield of all the vocational work,—great emphasis being placed on agriculture. The men were assigned quickly to the particular field that seemed best suited to their individual training and experience.

When the writer reached Paris, in the latter part of February, 1919, there were four classes of appointments being made: first, to the ports of embarkation; second, to Beaune University; third, to camps in France other than the ports; and fourth, to the Army of Occupation. The throng of workers in Paris had formed quite a definite opinion of the desirability of these several types of appointment. The ports were interesting, yet

disappointing, because the embarkation camps necessarily were always changing in personnel,—some divisions coming, some going. Under such conditions, educational work with any individual was limited to a period usually extending over two or three weeks.

Beaune University was a magnet that drew the majority of the men. The plans were so pretentious, the opportunity so large, that men selected for this service considered themselves very fortunate. Those called to duty at other stations in France were usually less pleased with the outlook, though many of them believed that nearness to the devastated areas made these assignments rather desirable.

The Army of Occupation was a much coveted assignment. Living conditions along the Rhine were reported attractive by the Third Army casuals on the way home and by those who came to Paris on furlough. It was reasonable also to establish schools for men who were to be stationed in definite posts for an extended period of time.

The Third Army arrived in the Rhinelands in December, 1918, after marching by several routes from their positions at the time of the armistice. Thus, the Second Division on the morning of November 11th was in action with the enemy. Part of it had crossed the Meuse near Beaumont, about 25 km. above Sedan, "in face of heavy concentrated enemy machine gun fire." The other divisions of the Third Army, also were actively engaged in the final campaign, several of them receiving special citations for "high courage, devotion to duty, and disregard for the immemorable hardships encountered. . . ." (General Order 26, Fifth Army Corps, Nov. 20th, 1918.)

A little more than a month after the armistice found the eight divisions in their several assigned locations along the Rhine, the 1st, 2d, and 32d, on the right, or east bank, and the 3d, 4th, 42d, 89th, and 90th on the left, or west bank,—over 200,000 American soldiers occupying about 2,500 square miles of German territory. Not many days after the battle-scarred divisions had been billeted in their German quarters, President Guy Potter Benton, of the University of Vermont, arrived in Coblenz, the army headquarters, to begin duty as regional educational director. In a short time Dr. Benton was assisted by some thirty

(*Concluded in next issue.*)

NEW BOOKS.

A Little Preserving Book for a Little Girl. By AMY L. WATERMAN. Boston, The Page Company. Pp. 197. $1.00.

The process of preserving and canning fruits and vegetables is told here in a simple way.

A Little Gardening Book for a Little Girl. By PETER MARTIN. Boston, The Page Company. Pp. 124. $1.00.

In concise form is given all that a little girl need know about the growing of the common flowers and vegetables, from the selection and preparation of the garden plot to the harvesting of the crop.

The Classroom Teacher. By GEORGE D. STRAYER and N. L. ENGELHARDT. New York, American Book Company. Pp. 400. $.

This book treats of the organization and administration of public education as well as of the technique employed by the teacher in his daily work. Among others there are interesting chapters on: Teaching Children to Study; Measuring the Achievements of Children; The Teacher and the Community.

Psychology for Teachers. By DANIEL W. LARUE. New York, American Book Company. Pp. 316. $.

The author's aim here is to present psychology psychologically and the method adopted consists in (1) passing from man in his environment to a study of "Body, Brain, and Mind," (2) showing the relation of adjustment between "The Mental and Environmental," and (3) explaining in the study of "Mind, Nervous System, and Behavior" how the neuromental governing and steering system accomplishes its complex task of adjustment.

Our Public Schools. By OSCAR T. CORSON. New York, American Book Company. Pp. 283.

A book full of good things.

Space and Time in Contemporary Physics. By MORITZ SCHLICK. New York, The Oxford Press. Pp. 89.

A valuable contribution to the theory of relativity and gravitation and will be welcomed by all interested.

Elementary Applied Mathematics. By W. PAUL WEBBER. New York, John Wiley and Sons. Pp. 115.

This book aims to provide a course that is complete in itself and which will meet the needs of those students who do not specialize in mathematics.

NOTES AND NEWS.

A Statement of the Plans and Purposes of the National Council of Teachers of Mathematics.

A realization of the need of a central organization to foster the interests of high school mathematics and to secure a greater degree of co-operation between individual teachers and between local associations of teachers interested in secondary school mathematics impelled a group of mathematics teachers to assemble at Cleveland last February at the time of the meeting of the Department of Superintendence of the N. E. A. There were present at this meeting 127 teachers of mathematics representing twenty states and as many local organizations. At that time The National Council of Teachers of Mathematics was formed. A constitution was adopted and the following officers elected:

President, C. M. Austin, Oak Park, Ill.

Vice-Pres., H. O. Rugg, New York City.

Sec.-Treas., J. A. Foberg, Chicago.

Other members of the Executive Committee: Marie Gugle, Columbus, Ohio; Jonathan Rorer, Philadelphia; W. D. Reeve, Minneapolis, Minn.; W. E. Beck, Iowa City, Iowa; W. A. Austin Fresno, Cal.; Harry Wheeler, Worcester, Mass.

A journal being absolutely necessary to the success of the council, the executive committee immediately began negotiations with the present management of The Mathematics Teacher looking to its adoption as the official organ of The National Council. These negotiations have resulted in action by the Association of Mathematics Teachers of the Middle States and Maryland, transferring the control of The Mathematics Teacher to the Council. The first number to appear under the new auspices will be the January issue.

Under the new management there will be eight numbers annually instead of four as at present, each number containing 48

pages of reading matter. The executive committee elected the following to be editors:

J. R. Clark, Lincoln School, New York City,
E. R. Smith, Baltimore, Md.,
H. D. Gaylord, Cambridge, Mass.,
Alfred Davis, St. Louis, Mo.,
J. W. Young, Dartmouth College.

A staff of contributing editors, fifteen in number, representing all sections of the country is soon to be chosen. In addition a live representative is desired in each local organization to report news items and to keep the Council in touch with the activities of these organizations. They will be able to report the results of any significant experiments or studies. The Council urges the fullest co-operation on the part of individual teachers.

We are mindful of the great influence and usefulness of THE MATHEMATICS TEACHER under the editorship of Dr. Metzler. The Council aims to continue this influence and usefulness, and to make the journal a force of nation-wide importance in the improvement of mathematics teaching in our secondary schools.

As the organ of the National Council of Teachers of Mathematics, THE MATHEMATICS TEACHER will attempt to render the following types of service to its readers:

1. Keeping them posted on what others are doing. Publicity will be given to the activities of the various local associations and to the work of individual teachers throughout the country. A spirit of fellowship will be developed among progressive teachers.

2. Provide a strong review department in which current publications will be promptly and carefully evaluated.

3. Offer its readers, through a Round Table section, a medium for the exchange of opinion and criticism. An opportunity to reply to any article or editorial which has appeared in the magazine will be offered in this section.

4. Publish only that type of article which will be helpful to the rank and file of class room teachers. Care will be exercised to select articles which describe successful class room practice, or which report investigations of interest to secondary school teachers. The selection of the editorial board from teachers

who are actually engaged in secondary school work indicates the desire of the National Council to make THE MATHEMATICS TEACHER a magazine which will promote the best thought and practice in this field.

Tentative Table of Contents of January Issue.

1. Organization and Plans of the National Council of Teachers of Mathematics .....President C. M. Austin.
2. The National Committee on Mathematical Requirements: A Report of Progress .....Professor J. W. Young.
3. Geometry in the Junior High School .....J. C. Brown.
4. The First Week in Demonstrative Geometry ..M. J. Newell.
5. Symposium: An Evaluation of the National Committee Report on Junior High School Mathematics by Ernst R. Breslich, William Betz, and W. W. Hart.

The executive committee of the Council heartily approves of the work of the National Committee on Mathematical Requirements and the journal will urge all teachers to study the committee's reports, and to modify their practice to conform to the committee's findings.

The journal will fully lend its columns to publicity of matter concerning the National Committee's activities and will at all times strive to aid in carrying forward the work begun by the National Committee.

Now, all this expenditure of time and money by the National Council will avail little unless we shall have the sincere co-operation of the individual teachers all over the country. If individuals and local organizations look on, merely waiting to see whether or not the Council succeeds before joining, then we are doomed to certain failure. On the other hand, if each individual mathematics teacher will do his share, success will be sure and swift. The Executive Committee of the Council is proceeding upon the assumption that there will be this hearty co-operation and that individuals and local organizations will make the fullest possible use of the journal and the Council.

C. M. AUSTIN.

CONSTITUTION OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS.

I. *Name.*

This organization shall be known as the National Council of Teachers of Mathematics,—those who are primarily interested in elementary and secondary mathematics.

II. *Objects.*

The purposes of this organization shall be,

1. To secure a greater degree of co-operation and solidarity among the teachers of mathematics.

(*a*) To provide for the wide publicity of important reports and addresses related to mathematics and the teaching of mathematics through an official organ and other publications.

(*b*) To vitalize and co-ordinate the work of the many organizations of mathematics teachers throughout the country.

2. To bring the interests of mathematics to the attention and consideration of the educational world.

III. *Time and Place of Meeting.*

Regular meetings of the Council shall be held annually in connection with the meeting of the Department of Superintendence of the National Education Association.

IV. *Membership.*

Membership in the Council shall be of two kinds: individual and collective.

All persons who are in sympathy with the work of the Council shall be eligible to individual membership.

All organizations of mathematics teachers shall be eligible to collective membership.

The annual dues of the individual members shall be $2.00.

The minimum dues for collective membership shall be three dollars, for each organization of less than fifty. For each additional hundred members, or fraction thereof, the dues shall be an additional five dollars.

V. *Management of the Council.*

1. Officers.

The officers of the Council shall be a President, a Vice-President, and a Secretary-Treasurer. Their duties shall be those commonly pertaining to these offices. The President and Vice-President shall be elected for a term of one year. The Secretary-Treasurer shall be elected for a term of three years.

2. Executive Committee.

There shall be an Executive Committee of nine members, three of whom shall be officers. The remaining six shall be elected, two each to hold office for a term of three years; except that at the first meeting, two shall be elected for one year, two for two years, and two for three years.

The Executive Committee shall manage the business of the Council, authorize the appointment of committees and fill vacancies in office.

3. Nominations.

At each annual meeting the President shall appoint a nominating committee of three members who shall report nominations for officers and for members of the executive committee.

VI. *Amendments.*

This Constitution may be amended by a two-thirds vote of the members present at any regular meeting.

YOU PUSH THE SLIDE. THE RULE DOES THE REST.

"Another raise in salary! You certainly are in luck, Jack."

"No luck about it, Bill; it's all due to this little thing called The Slide Rule.

"You know I've been working after school hours for an electrical manufacturing firm. A few days ago, our firm had a call on the 'phone from a contractor asking for an estimate on a big order of goods running into thousands of dollars. The contractor wanted an answer *while he held the wire* or he would give the job to some one else.

"My boss didn't know what to do about it. He couldn't figure out a reasonable estimate in so short a time and was about to tell the contractor so.

"I thought of my Slide Rule and said to the Boss: 'We can do it. Give me the number of articles of each kind and the price. My Slide Rule will do the rest.'

"Well, in less time than it takes to tell the story, we had the estimate ready and had closed the contract.

"But come outside. I want to show you a new out-drop I've been practicing. It's going to be a winner next week when I pitch against Central High School."

"Nothing doing. I have twenty examples in Physics to do and, believe me, they are some work."

"I had the same lesson, but I did those same examples in a few minutes by this same slide rule. You know almost all examples in Physics are solved by a proportion. Well! the slide rule gives you the answer to any proportion in a few seconds."

"And speaking of baseball, our scorer fixes up the batting average of every man on the team by using the slide rule. You see all you have to do is to divide the number of hits by the number of times at bat and get the result correct to three figures. This is done instantly by the slide rule."

"I say, Jack, that rule must be what my sister was talking about last evening. You know she is Private Secretary to the President of The Eastern Electric Company. She said that the reports which come to her office she checked by the slide rule in one third of the time that it formerly took. It's a wonder the High School wouldn't teach us the use of this rule. It would be worth much more than most of the stuff we get."

"Haven't you heard, Bill? The Mathematics Department has already ordered a supply of rules for regular use in the school. Mr. Smith, Head of the Department, says the slide rule is now recommended by a National Committee as a regular subject of study in all public high schools."

"Good work, I'll say. And I'll tell you what we'll do. If the rules don't come promptly, through the Board of Education, we'll form a Club for the study of the Slide Rule after school hours. Do you know where to send for rules and what about an instruction book?"

"Surely. Just write to Keuffel and Esser, 127 Fulton Street, New York City. They will give you prices and will send you their new self-teaching manual that anyone can understand."